Bose–Einstein effect in Z^0 decay and the weight method

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Abstract. We discuss the Bose–Einstein interference effect in multi-particle production. After a short review of various methods of implementation of this effect into Monte Carlo generators the weight method is presented in more detail and used to analyze the data for hadronic Z^0 decays. In particular, we consider the possibility of deducing the two-particle weight factor from the experimental data.

1 Introduction

In the last years many papers have been devoted to experimental and theoretical studies of Bose–Einstein interference effects [1] in multi-particle production. It has been argued that these studies may allow for the reconstruction of the space-time development of the interactions. In particular, different possibilities of implementing interference effects into Monte Carlo generators used for high energy hadroproduction processes were discussed. In this note we add some points to this discussion.

We present in detail some aspects of the weight method of implementing the BE effect in the MC generators. In particular, we try to establish to what extent one may reconstruct the two-particle weight function (related to the Wigner function) from the data on the BE effect. For this purpose we use the data on multi-particle production with the highest statistics available, the hadronic decays of Z^0's produced in e^+e^- collisions.

A short summary of various methods of implementing interference effects into Monte Carlo generators is presented in the next section. In the Sect. 3 we discuss the data from different LEP experiments on two-particle correlations from Z^0 hadronic decays which were used for analyzing the Bose–Einstein effect. Sect. 4 is devoted to the analysis of data in terms of the weight model. In particular, various choices of two-particle weight factors used in this method are compared. The last section contains some conclusions and an outlook for further investigations.

2 Implementation methods

The standard discussion of the BE effect in multi-particle production [2] starts from the classical space-time source emitting identical bosons with known momenta. Thus, the most natural procedure is to treat the original Monte Carlo generator as the model for the source and to symmetrize the final state wave function [3]. This may be done in a more proper way using the formalism of Wigner functions [4]. In any case, however, the Monte Carlo generator should yield both the momenta of the produced particles and the space-time coordinates of their creation (or last interaction) points. Even if we avoid troubles with the uncertainty principle by using the Wigner function approach, such a generator seems reliable only for heavy ion collisions. It has been constructed also for the e^+e^- collisions [5], but localizing the hadron creation point in the parton-based Monte Carlo program for lepton and/or hadron collisions is a rather arbitrary procedure, and it is hard to say what one really tests comparing such a model with the data.

It seems to be the best procedure to take into account the interference effects before generating events. Unfortunately, this was done until now only for the JETSET generator for a single Lund string [6–9], and a generalization for multi-string processes is not obvious. No similar modifications were yet proposed for other generators.

The most popular approach, applied since years to the description of BE effect in various processes, is to shift the final state momenta of the events generated by the PYTHIA/JETSET generators [10,11]. The prescription for a shift is such as to reproduce the experimentallly observed enhancement in the ratio

$$c_2(Q) = F \frac{\int d^3p_1 d^3p_2 \rho_2(p_1,p_2)\delta(Q-\sqrt{-(p_1-p_2)^2})}{\int d^3p_1 d^3p_2 \rho_1(p_1)\rho_1(p_2)\delta(Q-\sqrt{-(p_1-p_2)^2})}, \tag{1}$$

where F is a normalization factor:

$$F = \langle n \rangle^2 / \langle n(n-1) \rangle.$$

The function $c_2(Q)$ depends on a single invariant variable Q. The value of this function is close to one for the default JETSET/PYTHIA generator. One often parameterizes this ratio by

$$c_2(Q) = 1 + \lambda \exp(-R^2 Q^2), \tag{2}$$

where R and λ are parameters interpreted as the source radius and the "incoherence strength", respectively.

After performing the shifts, all the CM 3-momenta of the final state particles are rescaled to restore the original energy. In more recent versions of the procedure [12] "local rescaling" is used instead of the global one. In any case, each event is modified and the resulting generated sample now exhibits "BE enhancement": the ratio (1) is no longer close to one, and it may be parameterized as in (2).

There is no theoretical justification for this procedure, so it should be regarded as an imitation rather than implementation of the BE effect. Its success or failure in describing data is the only relevant feature. Unfortunately, whereas the method is very useful for the description of two-particle inclusive spectra, it fails to reproduce (with the same fit parameters R and λ) the three-particle spectra [13] and the semi-inclusive data [14]. This could certainly be cured, e.g., by modifying the shifting procedure and fitting the parameters separately for each semi-inclusive sample of data. However, the fitted values of parameters needed in the input factor (2) used to calculate shifts are quite different from the values one would get fitting the resulting ratio (1) to the same form [15]. This was shown recently in a much more detailed study [16]. Thus, it seems to be very difficult to learn something reliable of the space-time structure of the source from the values of the fit parameters in this procedure.

All this has lead to a revival of weight methods, which have been known for quite a long time [17], but were plagued with many practical problems. The method is clearly justified within the formalism of the Wigner functions, which allows one to represent (after some simplifying assumptions) any distribution with the BE effect built in as a product of the original distribution (without the BE effect) and the weight factor, depending on the final state momenta [18]. With the extra assumption of factorization in momentum space, we may write the weight factor for the final state with n identical bosons as

$$W(p_1, \ldots p_n) = \sum \prod_{i=1}^{n} w_2(p_i, p_{P(i)}), \qquad (3)$$

where the sum extends over all permutations $P_n(i)$ of n elements, and $w_2(p_i, p_k)$ is a two-particle weight factor reflecting the effective source size. A commonly used simple parameterization of this factor for a Lorentz symmetric source is

$$w_2(p, q) = \exp[(p - q)^2 / 2Q_0^2]. \qquad (4)$$

The only free parameter now is Q_0, representing the inverse of the effective source size. In fact, the full weight given to each event should be a product of factors (3) calculated for all kinds of bosons; in practice, pions of all signs are counted. Only direct pions and the decay products of ρ, K^* and Δ should be taken into account, since for other pairs a much larger R should be used, resulting in negligible contributions.

The main problem of the weight methods is that weights do change not only the Bose–Einstein ratio (1),

but also many other distributions. Thus, with the default values of the free parameters (fitted to the data without weights) we inevitably find some discrepancies with the data after introducing weights.

We want to make clear that this cannot be taken as a flaw of the weight method. There is no measurable world "without the BE effect", and it makes not much sense to ask if this effect changes e.g. the multiplicity distributions. If any model is compared to the data without taking the BE effect into account, the fitted values of its free parameters are simply not correct. They should be refitted with weights, and then the weights recalculated in an iterative procedure. This, however, may be a rather tedious task.

Therefore, we use a simple rescaling method proposed by Jadach and Zalewski [20]. Instead of refitting the free parameters of the MC generator, we rescale the BE weights (calculated according to the procedure outlined above) with a simple factor cV^n, where n is the global multiplicity of "direct" pions, and c and V are fit parameters. Their values are fitted to minimize

$$\chi^2 = \sum_n [cV^n N^w(n) - N^0(n)]^2 / N^0(n), \qquad (5)$$

where $N^0(n)$ is the number of events for the multiplicity n without weights, and $N^w(n)$ is the weighted number of events. This rescaling restores the original multiplicity distribution [24]. In addition, the single longitudinal and transverse momentum spectra are also restored by this rescaling [24].

Obviously, for a more detailed analysis of the final states, single rescaling may not be enough. For instance, since different parameters govern the average number of jets and the average multiplicity of a single jet, both should be rescaled separately to avoid a discrepancy with the data. Let us stress once again that such problems arise due to the use of generators with improperly fitted free parameters, and do not suggest any flaw of the weight method. Another problem is that our formula for the weights (3) is derived using some approximations, which are rather difficult to control [18]. We can justify them only a posteriori from the phenomenological successes of the weight method.

Last but not least, the main practical difficulty with (3) is the factorial increase of the number of terms in the sum with increasing multiplicity of identical pions n. For high energies, when n often exceeds 20, a straightforward application of (3) is impractical [19], and some authors [20,21] replaced it with simpler expressions, motivated by some models. It is, however, rather difficult to estimate their reliability.

We have recently proposed two ways of dealing with this problem. One method consists of a truncation of the sum (3) up to terms, for which the permutation $P(i)$ moves no more than five particles from their places [22]. However, it is difficult to claim a priori that such a truncation does not change the results which would be obtained using the full series (2).

Therefore a second way of an approximate calculation of the sum (2) was proposed [23]. Since this sum, called

the permanent of a matrix built from weight factors $w_{i,k}$, is quite familiar in field theory, one may use a known integral representation and approximate the integral by the saddle point method. However, this method is reliable only if in each row (and column) of the matrix there is at least one non-diagonal element significantly different from zero. Thus the prescription should not be applied to the full final states, but to the clusters, in which each momentum is not far from at least one other momentum. The full weight is then a product of weights calculated for the clusters, in which the state is divided.

The considerations presented above suggested the necessity of combining these two methods. After dividing the final state momenta of identical particles into clusters, we used for small clusters the exact formulae presented in [15, 22]. For large clusters (with more than five particles) we compared two approximations (truncated series and the integral representation) to estimate their reliability and the sensitivity of the final results to the method. Obviously, the results depend also on the clustering algorithm: if we restrict each cluster to particles very close in momentum space, the neglected contributions to the sum (3) from permutations exchanging pions from different clusters may be non-negligible, and if the cluster definition is very loose, the saddle point approximation may be unreliable. This was then also checked to optimize the algorithm used. We found that the truncated series approximation was sufficient in all cases we checked [24]. Our method has already been applied to the analysis of W^+W^- production [25].

3 The data and their analysis

Extremely high statistics collected in LEP for the hadronic decays of Z^0s produced in e^+e^- collisions allowed one to investigate in detail many interesting effects. In particular, the interference effects due to the Bose–Einstein statistics for pions were analyzed in several experiments.

The notorious difficulty in measuring the Bose–Einstein interference effects (BE effects) in multi-particle production is the proper choice of reference sample. In the early investigations the ratio of numbers of "like-to-unlike" charged pion pairs was analyzed as a function of three- or four-momentum difference squared,

$$R_{BE}(Q^2) = \frac{\int \rho_2^{++}(p_1, p_2)\delta[(p_1 - p_2)^2 + Q^2]d^3p_1 d^3p_2}{\int \rho_2^{+-}(p_1, p_2)\delta[(p_1 - p2)^2 + Q^2]d^3p_1 d^3p_2}. \quad (6)$$

Obviously, the denominator has Breit–Wigner peaks around the values of Q^2 corresponding to masses of resonances in the $\pi^+\pi^-$ system (and other peaks due to the maxima in mass spectra from three-pion resonances) which are absent in the numerator. To properly estimate the BE effects one should subtract these maxima, or to exclude the "resonance regions" from the Q^2 range used in fitting R_{BE} to the chosen function.

Therefore, recently a more common choice for the denominator was the "uncorrelated background" formed e.g. by choosing pairs of "like sign" pions from different events,

Table 1. Fits to the OPAL data

	R(fm)	λ	χ^2/d.o.f.
Reference fit to R_{BE}	0.955 ± 0.012	0.672 ± 0.013	402/40
Fit with Q bins of 50 MeV	0.962 ± 0.013	0.667 ± 0.014	204/17
Fit to R'_{BE}	0.793 ± 0.015	0.577 ± 0.010	185/40

which led to the definition presented in the former section, see (1). Here the main problem comes from neglecting the energy-momentum conservation effects (present in the numerator and absent in the denominator). However, for high energies and a restricted range of momenta (e.g. the "central region", often used for the analysis) such effects are expected to be rather small.

This method is easy to apply for hadronic or heavy ion collisions, where the initial momenta form the natural symmetry axis in the CM frame. For Z^0 decays the typical events are not aligned with the momenta of the initial e^+e^- pair. Thus the momenta chosen from different events should be rotated to the same symmetry axis before calculating Q^2. Unfortunately, such a procedure is not well defined: using sphericity, thrust or other variables one obtains different values of the rotation angles. Moreover, the three- and many-jet events do not have a well-defined symmetry axis, and limiting the analysis to the two-jet events would be rather arbitrary (and dependent on the jet definition).

Therefore, the best strategy seems to be using Monte Carlo generators, which are rather reliable for the process under discussion. Instead of analyzing the BE ratio R_{BE} one considers the "double ratio", i.e. the ratio of R_{BE}^{exp} given by the data and R_{BE}^{MC} computed from the MC generated events,

$$R'_{BE} = R_{BE}^{exp}/R_{BE}^{MC}. \quad (7)$$

An analogous "double ratio" has been defined also for the first definition of the "BE ratio" (1) [26]. The results obtained for two double ratios are inconsistent. In the following we use only the definition (7), preferred recently by the experimental groups [27,28].

The data with largest statistics have been presented by the OPAL group both for R'_{BE} and for R_{BE}. They compared the parameter values from the fits (with resonance regions excluded) to the function

$$f(x) = \kappa[1 + \lambda \exp(-R^2x^2)](1 + \alpha x^2). \quad (8)$$

The data (based on 3.6 million of events) were shown for $x = Q$ and for $x = q_t = ((\mathbf{p}_{1t} - \mathbf{p}_{2t})^2)^{1/2}$ [27]. We recall here in Fig. 1 the data and fits for $R_{BE}(Q)$ and $R'_{BE}(Q)$. The values of λ and R differ quite significantly, as shown in Table 1.

The other important conclusion drawn by the authors of that work was that even the fit for the double ratio
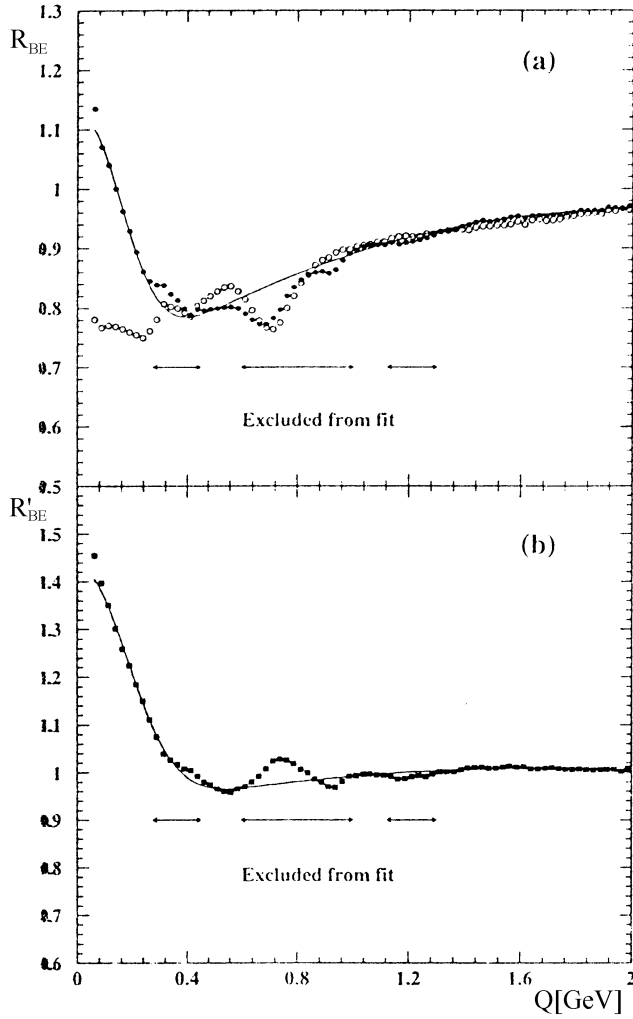
Fig. 1. a The BE ratio R_{BE} (6) from the OPAL data (black points) and from the JETSET MC without BE effect (open points); **b** the double ratio R'_{BE} (7) from the same data. Solid lines are fits to (8) [25]

Fig. 2. The BE double ratio R'_{BE} (7) from the ALEPH data. The solid line is a fit to (8) [28]

production in MC may distort the shape of the double ratio (2), making the detailed success or failure of the fits to specific functional forms rather ambiguous.

This seems to be the case for the recently presented ALEPH data for Z^0 decay used in the analysis of W pair decays [28]. As shown in Fig. 2, the data for R'_{BE} deviate from a smooth fit in a very similar way as the OPAL data shown in Fig. 1b (if one excludes the resonance ranges, the fit would go through points for Q between 0.4 and 0.6 GeV leaving the points below 0.4 GeV above the curve, and the excess around 0.7 GeV would be even more pronounced). Due to smaller statistics (below 100 000 events) much wider bins (of 0.1 GeV) were chosen.

Nevertheless, it seems that the claim of the authors of [28] that the data show a clear preference for one of the three models implementing the BE effect into MC requires a more careful analysis. In fact, if one would cut off the resonance regions (as done for OPAL data) certainly no conclusions could be drawn. If the full range of Q is used, one should ask before comparing models if they influence the size and/or shape of the resonance contributions to the BE ratios. We will later come back to these questions.

4 Weight methods; assumptions and results

Obviously, the Gaussian two-particle weight factor used in our method is just a simple ansatz. There are no deep reasons to expect such a shape (corresponding to a Gaussian distribution of two-particle birth-place position differences) for the weight factor. In fact, the Wigner functions (which serve to define the weight factors [18]) are not even necessarily positive. Thus, it is interesting not just to fit the Gaussian width to the data, but to investigate generally how the shape of R_{BE} (or R'_{BE}) depends on the parameters of the two-particle weight factors for their different functional forms. In principle the data may allow one to find the proper factor in momentum space and to deduce from that some information on the space-time structure of the source. For simplicity, we restrict ourselves here to the analysis of data presented in Sect. 3, and thus to the discussion of functions of a single variable Q^2 (which corresponds to the assumption of a space-time symmetric source).

requires cutting off the resonance regions. This is because the standard JETSET/PYTHIA MC generator does not satisfactorily describe the ratio R_{BE} not only for small Q (where the BE effect may appear), but also in the resonance region.

Let us add that one may interpret this disagreement as a signal that the production of K_S^0 and ρ is overestimated in this MC: the dips in experimental R_{BE} at $Q = 0.40$ and 0.72 GeV (corresponding to the K_S^0 and ρ masses, respectively) are less sharp in the data than in MC, which produces bumps in R'_{BE}.

In any case, the OPAL data show that one must be careful in interpreting the shape of the BE ratio (or double ratio) when using the unlike sign pairs as the reference sample. The fit parameters, which are tentatively interpreted as characterizing the source incoherence and geometrical size, depend significantly on the method of analysis. Moreover, the imperfect description of resonance
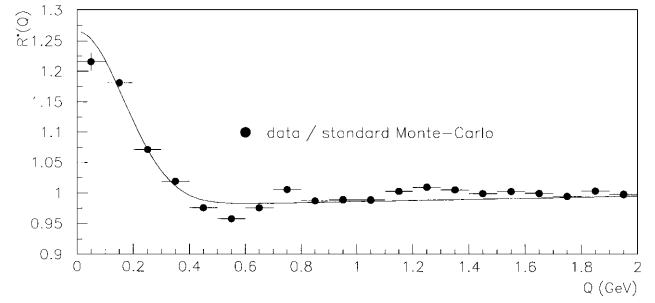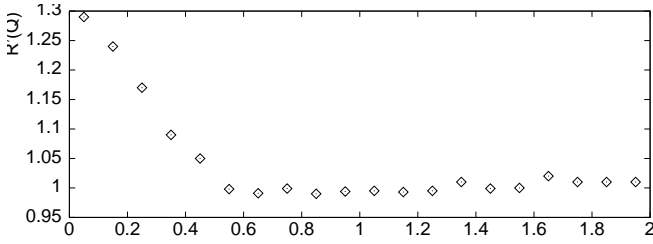
Fig. 3. The BE double ratio $R'_{\rm BE}$ (7) for an exponential weight factor with $Q_0 = 0.316$
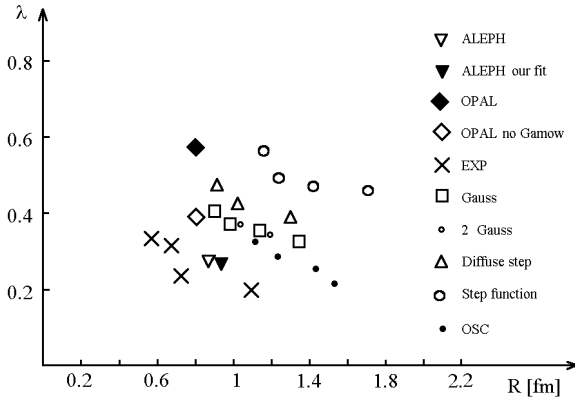

Fig. 4. The parameters of the fits to (2) for the first six bins of $R'_{\rm BE}$ from the samples of events generated with various weights factors: step function (circles), diffuse step (triangles), Gaussian (squares), double Gaussian (open points), exponent with oscillations (black points) and exponent (crosses). The parameters of the fits to the data are also shown as black diamonds (OPAL [27]), open diamonds (our fit to OPAL without Gamow correction), open triangles (ALEPH [28]) and black triangles (our fit to ALEPH)

We have performed MC generation of hadronically decaying Z^0s produced in the e^+e^- collisions at the peak energy with the standard PYTHIA/JETSET generator. Each event was assigned the weight calculated according to our method described in Sect. 2 with the two-particle weight factor given alternatively by a Gaussian (4), an exponent (e^{-Q/Q_0}), a step function (1 for $Q \leq Q_0$, 0 for $Q > Q_0$), a diffuse step function ($2/(1 + e^{Q/Q_0})$), a double Gauss ($\alpha e^{-Q^2/2Q_0^2} + (1-\alpha)e^{-Q^2/Q_0^2}$) or an oscillating exponent ($e^{-Q/Q_0} \sin(\alpha Q)/\alpha Q$) with various values of parameters for each form of the weight factor. For each set of parameters one million of events was generated.

For all samples the double ratio $R'_{\rm BE}$ was calculated for twenty equal bins in Q in the range $0 < Q < 2\,{\rm GeV}$. In all cases we found for small Q the values of $R'_{\rm BE}$ significantly to be above 1, and for $Q >> Q_0$ to be compatible with 1. An example (for an exponential weight factor with $Q_0 = 0.316$) is shown in Fig. 3.

A reasonable fit to (2) for the first few bins in Q was obtained in all the cases. We summarize all the results in Table 2 and in a simple two-dimensional diagram (Fig. 4), in which the fitted values of λ and R are given.

Table 2. Fits to MC results

Factor	Q_0[GeV]	α	λ	R[fm]	$R/R_{\rm eff}$
Step fct.	0.141		0.46	1.73	0.61
	0.158		0.47	1.44	0.57
	0.173		0.49	1.23	0.53
	0.187		0.56	1.16	0.54
Exponent	0.2		0.20	1.10	1.10
	0.244		0.24	0.73	0.90
	0.283		0.31	0.66	0.94
	0.316		0.33	0.57	0.90
Gauss	0.2		0.33	1.34	0.75
	0.224		0.36	1.15	0.72
	0.245		0.37	0.99	0.68
	0.265		0.41	0.89	0.66
Diff. step	0.172		0.39	1.30	0.73
	0.2		0.42	1.02	0.66
	0.224		0.47	0.90	0.65
2 Gauss	0.2	0.2	0.34	1.19	0.74
	0.2	0.5	0.37	1.04	0.73
Osc. exp.	0.244	8	0.21	1.53	0.69
	0.316		0.25	1.43	0.65
	0.448		0.28	1.23	0.55
	0.632		0.32	1.12	0.49

Each symbol in the diagram corresponds to a given form of the weight factor. Increasing values of Q_0 correspond always to the increasing values of λ and decreasing values of R. The values fitted by the OPAL and ALEPH collaborations to their data are also shown. Since these fits are performed to more complex formulae (e.g. (8)) and in different ranges of Q, we have checked that very similar values result from our simple fits. We will comment later on other LEPII experiments.

Q_0 is not the best measure of the "width in momentum space" when one compares different shapes of the weight factor. It seems more natural to use $\overline{Q}$, which is just the average value of Q for a given weight factor:

$$\overline{Q} = \frac{\int_0^\infty Q w_2(Q)\mathrm{d}Q}{\int_0^\infty w_2(Q)\mathrm{d}Q}. \tag{9}$$

If we define $R_{\rm eff} = 1/\overline{Q}$ as the "effective source size" we find a close relation between the fitted values of R and

the input values of R_{eff}. The ratio R/R_{eff}, quoted in the last column of Table 2, is in the range 0.5–1 for almost all values of Q_0 and all shapes of the weight factor.

The first obvious conclusion from Fig. 4 is that the fits to the two experimental sets of data are strongly different. This is partly due to the Gamow correction for the effect of electromagnetic final state interactions, which was used by OPAL and neglected by ALEPH. Since it has recently been argued that this correction badly overestimates the effect [29], we have also shown an estimate for the OPAL point without Gamow correction (obtained using the values for correction quoted in [27]). Now the two data points are much closer, but still they differ significantly.

The values resulting from our generated samples cover quite a wide range both in λ and R. Certainly, it is possible to reproduce any of the experimental values by a suitable choice of the form and free parameters of our weight factors. However, we do not think this would give a valuable information on the "proper" choice of the weight factor reflecting the "real" space-time structure of the source.

There are a few reasons for such scepticism. First, as noted in the previous section, the shape of the "experimental" R'_{BE} strongly depends on the quality of the "reference MC". Since we know that already at $Q \cong 0.40\,\text{GeV}$ the influence of the (poorly described) K_S^0, η and η' contributions distorts the shape, the fitted values of R around $1\,\text{fm} \cong 0.2\,\text{GeV}^{-1}$ are not very reliable. The situation is even less clear if one fits a wider range of Q using a more elaborate ansatz for R'_{BE} (e.g. (8)). Here the choice of the "cut out resonance region" may quite significantly influence the fit results. Moreover, the method of implementing the BE effect may influence the resonance contribution. For our method the absence of peaks or bumps in the double ratio for the "MC with weights" over the "MC without weights" (as seen, e.g., in Fig. 3) suggests that this is not the case (the oscillations shown in Fig. 2 of [28] may result from lower statistics). This should also be checked for any other method.

Second, the choice of "direct" pions affected by the BE effects described in Sect. 2 (although the same in our method and in the momentum shift method of [10–12]) is by no means unique. In fact, even for the fastest decaying resonances the effective "source radius" for the decay products should not be exactly the same as for the "real direct pions". Thus the choice of a single value of Q_0 in our weight factor is certainly an oversimplification. The total exclusion of the decay products of long-living resonances is also an approximation: some of them may be born quite close to the collision point and contribute to the visible BE effect. The selection and treatment of "direct" pions included in calculating weights (or subject to the momentum shift in the method of [10–12]) strongly influences the resulting value of λ.

Third, the standard fitting method of BE ratios or double ratios is also a subject of criticism [30]. More plausible methods suggest larger uncertainties of the fit parameters.

The situation would not improve if we add the other published data, which used much lower statistics [31] or a different form for the fitted function [32]. One may in short

summarize the situation by stating that it is too early to deduce details of the source space-time structure from the existing data, even if we assume the applicability of all the assumptions used in formulating the weight method.

Another interesting feature is the relative stability of our results with respect to the detailed shape of the two-particle weight factor. The fitted values of λ fall in the range of values 0.2–0.5 for all the functions considered. They are highest for the step function, lower for "diffuse step function", still lower for a Gaussian and lowest for the exponent. One may summarize that for the same "half width" of the weight factor function one gets a higher value of λ if the function stays longer near the value of 1 at small Q. As already noted, λ depends much stronger on the percentage of pions counted as "direct". The values of λ below 1 mainly reflect the percentage of direct like-sign pion pairs, which was already observed when using Sjöstrand's method of BE effect implementation [32].

A new result is the observed relative insensitivity of the results on the oscillations in the two-particle weight factor. In fact, the oscillation quarter-period $\pi/(2\alpha) \approx 0.2\,\text{GeV}$ serves as an effective range of correlations when it is smaller than Q_0. Therefore, R_{eff} is almost the same for all the values of Q_0 quoted in Table 2. The non-positive oscillating values of weight factor for larger Q do not result in values of R'_{BE} oscillating around 1, as one could naively expect; averaging over many pairs in each event and over many events kills any trace of the oscillations. This is quite important, as many simple forms for the weight factor in space-time (e.g. the step function) result in an oscillating form in momentum space.

On the other hand, there is an obvious dependence of the fitted values of R on the parameter Q_0 (or R_{eff}), which determine how fast the weight factor decreases with increasing Q. R/R_{eff} has quite similar values for various shapes of the weight factors and various values of Q_0. Thus in our method R reflects in some sense the "source size", as expected. As already noted, this is true also for the oscillations in momentum space. Increasing Q_0 one increases also the fitted values of λ, but this effect is weaker.

5 Conclusions and outlook

We have analyzed the data on the BE effect in multi-hadron states obtained from Z^0 decays within the framework of the weight method of implementing this effect. We show that the resonance contributions are not satisfactorily described by the standard MC generators, which makes the analysis of "double ratio" data quite difficult. In particular, the choice of the "proper" two-particle weight factor which would describe the data best is obviously dependent on the way the data are processed. Thus one should improve (if possible) the quality of the MC generators before drawing any serious conclusion on the agreement or disagreement between the models and data on the BE effect. The alternative is to elaborate better methods of preparing the "reference sample" using only the like-sign pairs, which are much less affected by the resonance effects.

On the other hand, we found that the analysis makes no great difference between the positively defined and oscillating weight factors in momentum space, provided they correspond to a similar "half-width". This means that the results are not too strongly dependent on the "sharpness of the source boundary" in space-time; one may hope to recover the proper "source size" in our method.

Let us stress that the analysis presented in this paper is just the first step: We do not discuss the anisotropy of the source nor the time dependence reflected in the BE effect. Also, we do not consider here the possibilities of comparing the effect for the selected classes of events (defined i.e. by multiplicity, by the event shape parameters or by some special triggers). In our opinion, the weight method is well suited to discuss these subjects and we hope to consider them in the near future.

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